3-D Geologic Modeling and Visualization of Geologic Boundaries: Theory Based on The Generalized Geologic Function

Go Yonezawa, Shinji Masumoto and Kiyoji Shiono
Department of Geosciences, Graduate School of Science, Osaka City University, 3-3-138 Sugimoto, Sumiyoshi-ku, Osaka 558-8585, Japan
Tel: +81-6-6605-2594     Fax: 81-6-6605-3071     E-mail: goy@sci.osaka-cu.ac.jp

Abstract

The geologic function that assigns a unique geologic unit to every point in the objective three-dimensional (3-D) space is a key element of a computerized geo-mapping. Algorithms for construction and visualization of 3-D geologic models based on the geologic function have been widely developed. As the concept of geologic boundary is not contained in the geologic function, we newly define the generalized geologic function that assigns a pair of right above and right below geologic units to every point in the objective 3-D space. The generalized geologic function clarifies a boundary between geologic units to be visualized. Visual Basic program Geomodel2003 was developed in order to visualize geologic boundaries on the objective surface by embedding sub-routines for visualization of geologic boundary that had been developed in Geomodel2000. We verified the utility of the previously proposed algorithm. The Application of Geomodel2003 to a test data in the Honjyo area, Akita Prefecture, Japan, proved that the proposed algorithm is valid for 3-D geologic modeling.

1. Introduction

It is necessary to carry out two steps to draw a geologic map from a collection of observations, (1) drawing a geologic boundary of geologic units on topographic map, (2) painting areas which are enclosed by geologic boundaries with proper colors of geologic units. The method of three-dimensional (3-D) geologic modeling based on the logical model of geologic structure has been developed by Sakamoto et al. (1993), Masumoto et al. (1997) and Shiono et al. (1998), and its actual visualization of 3-D geologic model has been proposed by Masumoto et al. (1999) using GRASS GIS and Sakamoto et al. (2000) using Visual Basic program Geomodel 2000. It is possible to draw the 2-D geologic map, the vertical geologic section and the 3-D geologic map. However, there is still unsolved matter with regard to visualization of geologic boundaries, which is one of the most important elements of any geologic map. To solve the problem Masumoto et al. (2001) and Yonezawa et al. (2002) proposed that there is a capability for drawing a geologic boundary when we determine the geologic units at grid points of four corners using the geologic function in Geomodel2000. However, there is a problem that nonexistent geologic boundaries may display when the grid distance is expanded. Although such a problem can be avoided practically when the grid distance is diminished, it is still inadequate as a theory for drawing geologic boundaries.

We propose the generalized geologic function which is improved on the geologic function to adjust the problem. In our approach, we thought that it is most important to define the generalized geologic function. It shows that geologic a boundary can be drawn in the geologic...
geologic units $b_0$, $b_1$, and $b_2$ are expressed as follows:

\begin{align*}
  b_1 &= S_1^- \cap S_2^- \quad \text{Equation 3} \\
  b_2 &= S_1^+ \cap S_2^- \quad \text{Equation 4} \\
  b_0 &= S_2^+ \quad \text{Equation 5}
\end{align*}

The distribution of geologic units $b_0$, ..., $b_n$ are defined by surfaces $S_1, ..., S_q$. The logical relation between the distribution of geologic units and the surfaces is termed the logical model of geologic structure.

2.2 Geologic Function

As the geologic units $b_0, b_1, ..., b_n$ are defined by surfaces, they can be expressed in minset standard forms (Gill, 1976). The minset is a minimum subspace divided by the surfaces $S_1, ..., S_q$ in the space $\Omega$. Each minset defined by:

\[ m_{d_1 \ldots d_q} = X_1 \cap X_2 \cap ... \cap X_q \quad \text{Equation 6} \]

where $X_k = \begin{cases} 
  S_k^- ; & d_k = 0 \\
  S_k^+ ; & d_k = 1 
\end{cases} \quad (k = 1, 2, ..., q)$

For example, two surfaces $S_1$ and $S_2$ generate four minsets as follows:

\begin{align*}
  m_{00} &= S_1^- \cap S_2^- \quad \text{Equation 7} \\
  m_{01} &= S_1^+ \cap S_2^- \quad \text{Equation 8}
\end{align*}

According to Shiono et al. (1994; 1998), logical models of geologic structure, geologic function and minset are explained simply as a preparation to introduce the generalized geologic function.

2.1 Logical Model of Geologic Structure

Let a 3-D space $\Omega$ be an objective survey area and suppose that the space $\Omega$ is divided into two subspaces by a surface $S$. $S^+$ and $S^-$ give subspaces that lie above and below the surface $S$, respectively (Figure 1). The surface $S$ is contained in subspace $S^-$. Then we have the following relations:

\begin{align*}
  S^+ \cup S^- &= \Omega \quad \text{Equation 1} \\
  S^+ \cap S^- &= \emptyset \quad \text{Equation 2}
\end{align*}

The space $\Omega$ is composed of $n$ geologic units $b_1, ..., b_n$ and open space $b_0$ (air). Figure 2 shows a simple geologic structure in the vertical section. The geologic unit $b_1$ represents basement rocks. After sedimentation and erosion, the geologic unit $b_2$ is formed. Surface $S_1$ is a geologic boundary surface, and surface $S_2$ is a topographic surface. The distribution of

![Figure 1: The objective space $\Omega$ is divided into two subspaces on surface](image1.png)

![Figure 2: The simple geologic structures](image2.png)
When the logical model of geologic structure is given, the distribution of geologic units \( b_0, \ldots, b_n \) can be expressed in a union of minsets generated by the surfaces \( S_1, \ldots, S_q \). In the case of the geologic structure shown in Figure 2, the geologic units are expressed as follows:

\[
\begin{align*}
    b_0 &= (S_1 \cup S_2^-) \cap S_2^+ = m_{01} \cup m_{11} \\
    b_1 &= m_{00} \\
    b_2 &= m_{10}
\end{align*}
\]

Then we can find the relation between a set of minset \( M = \{ m_{00}, m_{01}, m_{10}, m_{11} \} \) and a set of geologic units \( B = \{ b_0, b_1, b_2 \} \) as follows:

\[
\begin{align*}
    m_{00} &\subset b_1 \\
    m_{01} &\subset b_0 \\
    m_{10} &\subset b_2 \\
    m_{11} &\subset b_0
\end{align*}
\]

Let \( g_f: M \rightarrow B \) be a function that assigns every minset to a geologic unit that contains the minset. Table 1 gives the function for \( g_f \) the geologic structure shown in Figure 2. Further, for a point \( P(x, y, z) \) in a space \( \Omega \), a minset \( m_d \) can be assigned a value of \( d_k = 0 \) or \( d_k = 1 \) depending on whether \( P(x, y, z) \) lies in \( S_k^+ \) or \( S_k^- \), respectively. This correspondence between every point in the space \( \Omega \) and minset is expressed by a function \( g_2: \Omega \rightarrow M \). Therefore, the function \( g: \Omega \rightarrow B \) is expressed to compound the function \( g_1 \) and function \( g_2 \) as follows:

\[
g(x, y, z) = g_1(g_2(x, y, z))
\]
3.1 Right Above Minset and Right Below Minset

When minset \( m_0 \) and \( m_1 \) are generated by the surface \( S \) which divides space \( \Omega \) into two subspaces, point \( P (x, y, z) \) on surface \( S \) is included in \( m_0 \) by definition (Figure 4). Point \( P' (x, y, z-\varepsilon) \) lies lower than point \( P (x, y, z) \) by infinitesimal distance \( \varepsilon \) (>0) is included in \( m_0 \) as well. The minset including point that lies lower than point \( P (x, y, z) \) by an infinitesimal distance is termed the right below minset of point \( P \). Point \( P'' (x, y, z+\varepsilon) \) that lies above point \( P (x, y, z) \) by an infinitesimal distance is included in \( m_1 \). The minset that includes point that lie above point \( P (x, y, z) \) by an infinitesimal distance is termed the right above minset of point \( P \).

It can be generally explained by minset that are generated by the surfaces \( S_1, \ldots, S_q \), which divides the space \( \Omega \) into two subspaces. When the concrete properties of surfaces are shown, character string \( c_1, \ldots, c_q \) is defined by:

\[
\begin{align*}
0 & ; \text{Point } P \text{ lies lower than } S_k \\
* & ; \text{Point } P \text{ lies on } S_k \\
1 & ; \text{Point } P \text{ lies higher than } S_k
\end{align*}
\]

Equation 19

When point \( P \) does not exist on surface \( S \), character string is composed of binary number, 0 and 1. When point \( P \) exists on surface \( S \), character string include *.

When point \( P \) exists on plural surfaces, it is included plural * in the character string. Thus, we can introduce the concept of right above and right below minset of point \( P \). It is defined generally as follows:

Let character string be \( c_1, \ldots, c_q \). When all * are replaced with 0, it is considered that the character string is binary number. The minset which represents binary subscript corresponds to that character string is termed the right below minset of point \( P \).

When all * are replaced with 1, it is considered that the character string is binary number. The minset which represents binary subscript corresponds to that character string is termed the right above minset of point \( P \).

Therefore, there is defined a function \( g_2' : \Omega \rightarrow M \times M \) that assigns a pair of the directly above and below minsets \((m, m')\) to every point \( P (x, y, z) \) in the objective 3-D space \( \Omega \). Next, we explain an example of execution to obtain a pair of directly above and below minsets \((m, m')\).

It can be shown that the five points \( P_1, \ldots, P_5 \) correspond to values of \( g_2' \). In the case of point \( P_1 (x, y, z) \) that lies lower than both surfaces \( S_1 \) and \( S_2 \), \( c_1^2 = 00 \).
In the case of a point \( P_2 (x_2, y_2, z_2) \) on surface \( S_1 \) and lower than surface \( S_2 \), \( c_1c_2 \) is 0. When \( * \) interchanges 0 and 1, it gets two character strings 00 and 10:

\[
g_2' (x_2, y_2, z_2) = (m_{00}, m_{10})
\]  
Equation 21

In the case of a point \( P_3 (x_3, y_3, z_3) \) that lies higher than surface \( S_1 \) and on surface \( S_2 \), \( c_1c_2 \) is 1. When \( * \) interchanges 0 and 1, it gets two character strings 10 and 11:

\[
g_2' (x_3, y_3, z_3) = (m_{10}, m_{11})
\]  
Equation 22

In the case of a point \( P_4 (x_4, y_4, z_4) \) on surface \( S_1 \) and that lies higher than surface \( S_2 \), \( c_1c_2 \) is 1. When \( * \) interchanges 0 and 1, it gets two character strings 01 and 11:

\[
g_2' (x_4, y_4, z_4) = (m_{01}, m_{11})
\]  
Equation 23

In the case of a point \( P_5 (x_5, y_5, z_5) \) on surface \( S_1 \) and surface \( S_2 \), \( c_1c_2 \) is **. When \( * \) interchanges 0 and 1, it gets two character strings 00 and 11:

\[
g_2' (x_5, y_5, z_5) = (m_{01}, m_{11})
\]  
Equation 24

3.2 Generalized Geologic Function

Every point \( P \) in the space \( \Omega \) corresponds to a pair of minset \( (m, m') \) by function \( g_2' : \Omega \rightarrow M \times M \). Therefore, minset \( m \) lies right below point \( P \) corresponds to geologic unit \( g_1 (m) \) including the point by function \( g_1 : M \rightarrow B \). Such geologic unit is termed “the right below geologic unit of point \( P \)”. Minset \( m' \) lies right above point \( P \) is corresponding to geologic unit \( g_1 (m') \) including the point by function \( g_1 \) as well. Such a geologic unit is termed “the right above geologic unit of point \( P \)”.

Through this method, it is possible to define the function \( g' : \Omega \rightarrow B \times B \) that corresponds to a pair of geologic units that lie in both right above and below the point \( P \). The function \( g' \) is termed the generalized geologic function. When a pair of minset \( (m, m') \) corresponds to a pair of geologic units \( (g_1(m), g_1(m')) \) by function \( g_1 : M \times M \rightarrow B \times B \), the generalized geologic function \( g' \) is possible to define by:

\[
g'(x, y, z) = g_1' (g_2'(x, y, z))
\]  
Equation 25

It can be shows that the five points \( (P_1, ..., P_5) \) of Figure 3 corresponds to value of \( g' \) in Figure 5. In the case of a point \( P_1 (x_1, y_1, z_1) \) that lies lower than both surfaces \( S_1 \) and \( S_2 \):
3.3 Determiner Algorithm of Minset Right Below and Right Above the Optional Point

The processing to determine the character string and the processing to substitute * for 1 or 0 can be done all together as follows, yet it is necessary to let the elevation of surface $S_1, \ldots, S_q$ dividing space $\Omega$ into two, upper and lower be $z_k = s_k(x, y)$ ($k = 1, \ldots, q$) and let the difference between the elevation $H$ of point $P$ and $z_k$ be $D_k = H - z_k$.

1. $d_k (k = 1, \ldots, q)$ are given by:
   
   \[
   \begin{cases}
   D_k > 0 : d_k = 1 \\
   D_k = 0 : d_k = 0 \\
   D_k < 0 : d_k = 0
   \end{cases}
   \]

   Equation 33

   The $q$ digit binary number $d_1'd_2'\ldots d_q'$ represent the index of minset right below point $P$.

2. $d_k' (k = 1, \ldots, q)$ are given by:

   \[
   \begin{cases}
   D_k > 0 : d_k' = 1 \\
   D_k = 0 : d_k' = 1 \\
   D_k < 0 : d_k' = 0
   \end{cases}
   \]

   Equation 34

   The $q$ digit binary number $d_1'd_2'\ldots d_q'$ represent the index of minset right above point $P$.

Minset right below and right above point $P$ can be determined by Equations 33, 34, yet it is necessary to pay attention, when we introduce this method into computer processing. Numerical calculation by the computer is approximate calculation of finite numbers of digit. Since it should not be $D_k = 0$ when we deal with difference $D_k$ between point $P$ and surface $S_k$ as real number. Point $P$ should be a point on the surface when point $P (x, y, z)$ lies within the elevation $z$ of surface in the range $\pm \varepsilon$ (Figure 6). In this case, Equations 33, 34 become as follows:

1'. $d_k' (k = 1, \ldots, q)$ are given by:

   \[
   \begin{cases}
   D_k > \varepsilon : d_k' = 1 \\
   \left| D_k \right| = \varepsilon : d_k' = 1 \\
   D_k < -\varepsilon : d_k' = 0
   \end{cases}
   \]

   Equation 33'.

   The $q$ digit binary number $d_1'd_2'\ldots d_q'$ represent the index of minset right below point $P$.

2'. $d_k' (k = 1, \ldots, q)$ are given by:

   \[
   \begin{cases}
   D_k > \varepsilon : d_k' = 1 \\
   \left| D_k \right| = \varepsilon : d_k' = 1 \\
   D_k < -\varepsilon : d_k' = 0
   \end{cases}
   \]

   Equation 34'

   The $q$ digit binary number $d_1'd_2'\ldots d_q'$ represent the index of minset right above point $P$.


We added algorithms to distinguish the geologic boundary using the generalized geologic function and produced Visual Basic program Geomodel2003. Examples of the various geologic maps are presented in Figure 7 using Geomodel2003. The study area ($8.7 \times 6.5$ km) is located in the Honjo region of Akita Prefecture, Northeast Japan using data extracted from a geologic map (Osawa et al., 1977). The mapped district is underlain by Neogene rocks and Quaternary alluvium. The Neogene formations, 3,000 m to 5,000 m in total thickness, consist mainly of sedimentary rocks with acid tuff. The main part of this area is characterized by intense folds with a general trend in the N-S direction. The Quaternary alluvial deposits unconformably overlay the Neogene formations, and are widely distributed along rivers.

Figure 6: Determination of minset right below and right above the optional point.
sections. The distribution of geologic units is shown in Figure 7(a) as the 3-D geologic map. Figure 7(b) is the 3-D geologic map in the case of including the geologic boundaries. Figure 7(c) is the 2-D geologic map. Figure 7(d) is the vertical geologic section map on the line (A)–(B) in Figure 7(c).

5. Conclusion

We defined the generalized geologic function which corresponds to a pair of geologic units laying both right above and right below the every point in the objective 3-D space based on the geologic function. It is possible to distinguish the geologic boundary by applying the generalized geologic function to points on surfaces. We developed the theory and algorithms to visualize the geologic boundary, and through this concept produced Visual Basic program Geomodel2003. As we improved the sub-routine which had been developed before, the geologic boundaries can resultantly be visualized on various geologic maps. There should be considerable potential to represent geologic boundaries in the GRASS GIS, when this method is introduced to GRASS GIS.

Acknowledgment

This research was partially supported by the Japanese Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Scientific Research (C) (2), Project No. 14540430.

Figure 7: Example of the various geologic maps using Geomodel2003, (a), (b) the 3-D geologic map (in the case of including the geologic boundaries and not.), (c) the 2-D geologic map, (d) the vertical geologic section map on (c)
References


Masumoto, S., Raghavan, V., Aoyama, T. and Shiono, K., 1999, Three dimensional geologic modelling on GIS using the data from geological sheet map -A case study in Ojiya district, Niigata Prefecture, Japan-, Geoinformatics, 10(2), 96-99.


Sakamoto, M., Shiono, K. and Masumoto, S., 2000, Practical solution of 3-D geomapping system based on geology oriented logical system, Geoinformatics, 11(2), 116-117.

