3-D Geologic Modeling and Visualization of Geologic Boundaries: Theory Based on The Generalized Geologic Function

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Abstract

The geologic function that assigns a unique geologic unit to every point in the objective three-dimensional (3-D) space is a key element of a computerized geo-mapping. Algorithms for construction and visualization of 3-D geologic models based on the geologic function have been widely developed. As the concept of geologic boundary is not contained in the geologic function, we newly define the generalized geologic function that assigns a pair of right above and right below geologic units to every point in the objective 3-D space.
The generalized geologic function clarifies a boundary between geologic units to be visualized. Visual Basic program Geomodel2003 was developed in order to visualize geologic boundaries on the objective surface by embedding sub-routines for visualization of geologic boundary that had been developed in Geomodel2000. We verified the utility of the previously proposed algorithm. The Application of Geomodel2003 to a test data in the Honjyo area, Akita Prefecture, Japan, proved that the proposed algorithm is valid for 3-D geologic modeling.

1. Introduction

It is necessary to carry out two steps to draw a geologic map from a collection of observations, (1) drawing a geologic boundary of geologic units on topographic map, (2) painting areas which are enclosed by geologic boundaries with proper colors of geologic units. The method of three-dimensional (3-D) geologic modeling based on the logical model of geologic structure has been developed by Sakamoto et al. (1993), Masumoto et al. (1997) and Shiono et al. (1998), and its actual visualization of 3-D geologic model has been proposed by Masumoto et al. (1999) using GRASS GIS and Sakamoto et al. (2000) using Visual Basic program Geomodel 2000. It is possible to draw the 2-D geologic map, the vertical geologic section and the 3-D geologic map. However, there is still unsolved matter with regard to visualization of geologic boundaries, which is one of the most important elements of any geologic map. To solve the problem Masumoto et al. (2001) and Yonezawa et al. (2002) proposed that there is a capability for drawing a geologic boundary when we determine the geologic units at grid points of four corners using the geologic function in Geomodel2000. However, there is a problem that nonexistent geologic boundaries may display when the grid distance is expanded. Although such a problem can be avoided practically when the grid distance is diminished, it is still inadequate as a theory for drawing geologic boundaries.

We propose the generalized geologic function which is improved on the geologic function to adjust the problem. In our approach, we thought that it is most important to define the generalized geologic function. It shows that geologic a boundary can be drawn in the geolo-

ISSN 1686-6576/© Geoinformatics International

gic map using the generalized geologic function. Further, we verified the utility of the proposed algorithm previously. The example of application for the geologic map including the geologic boundaries is shown by using Geomodel 2003.

2. Basic Theory

According to Shiono et al. (1994; 1998), logical models of geologic structure, geologic function and minset are explained simply as a preparation to introduce the generalized geologic function.

2.1 Logical Model of Geologic Structure

Let a 3-D space Ω be an objective survey area and suppose that the space Ω is divided into two subspaces by a surface S. S^+ and S^- give subspaces that lie above and below the surface S, respectively (Figure 1). The surface S is contained in subspace S^-. Then we have the following relations:

$$S^+ \cup S^- = \Omega \qquad \text{Equation 1}$$

$$S^+ \cap S^- = \varnothing \qquad \text{Equation 2}$$

The space Ω is composed of n geologic units $b_1, \ldots, b_n$ and open space b_0 (air). Figure 2 shows a simple geologic structure in the vertical section. The geologic unit b_1 represents basement rocks. After sedimentation and erosion, the geologic unit b_2 is formed. Surface S_1 is a geologic boundary surface, and surface S_2 is a topographic surface. The distribution of geologic units b_1, b_2 and b_0 are expressed as follows:

$$b_1 = S_1^- \cap S_2^- \qquad \text{Equation 3}$$

$$b_2 = S_1^+ \cap S_2^- \qquad \text{Equation 4}$$

$$b_0 = S_2^+ \qquad \text{Equation 5}$$

The distribution of geologic units $b_0, \ldots, b_n$ are defined by surfaces $S_1, \ldots, S_q$. The logical relation between the distribution of geologic units and the surfaces is termed *the logical model of geologic structure*.

2.2 Geologic Function

As the geologic units $b_0, b_1, \ldots, b_n$ are defined by surfaces, they can be expressed in *minset standard forms* (Gill, 1976). The minset is a minimum subspace divided by the surfaces $S_1, \ldots, S_q$ in the space Ω. Each minset defined by:

$$m_{d_1 d_2 \ldots d_q} = X_1 \cap X_2 \cap \ldots \cap X_q \qquad \text{Equation 6}$$

$$\text{where } X_k = \begin{cases} S_k^- ; \ d_k = 0 \\ S_k^+ ; \ d_k = 1 \end{cases}$$

$$(k = 1, 2, \ldots, q)$$

For example two surfaces S_1 and S_2 generate four minsets as follows:

$$m_{00} = S_1^- \cap S_2^- \qquad \text{Equation 7}$$

$$m_{01} = S_1^- \cap S_2^+ \qquad \text{Equation 8}$$

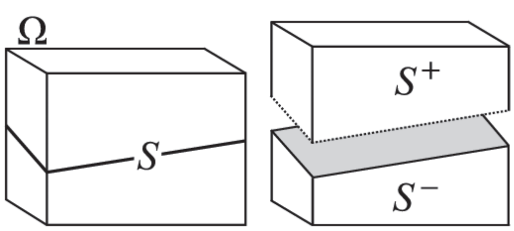

Figure 1: The objective space Ω is divided into two subspaces on surface

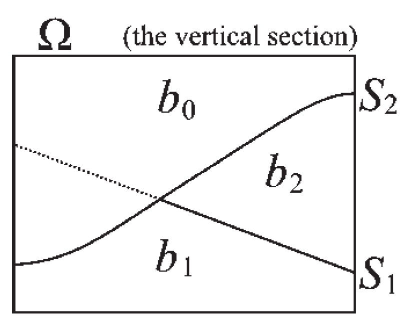

Figure 2: The simple geologic structures

$$m_{10} = S_1^+ \cap S_2^- \quad \text{Equation 9}$$

$$m_{11} = S_1^+ \cap S_2^+ \quad \text{Equation 10}$$

When the logical model of geologic structure is given, the distribution of geologic units b_0, …, b_n can be expressed in a union of minsets generated by the surfaces S_1, …, S_q. In the case of the geologic structure shown in Figure 2, the geologic units are expressed as follows:

$$b_0 = (S_1^+ \cup S_1^-) \cap S_2^+ = m_{01} \cup m_{11} \quad \text{Equation 11}$$

$$b_1 = m_{00} \quad \text{Equation 12}$$

$$b_2 = m_{10} \quad \text{Equation 13}$$

Then we can find the relation between a set of minset $M = \{ m_{00}, m_{01}, m_{10}, m_{11} \}$ and a set of geologic units $B = \{ b_0, b_1, b_2 \}$ as follows:

$$m_{00} \subset b_1 \quad \text{Equation 14}$$

$$m_{01} \subset b_0 \quad \text{Equation 15}$$

$$m_{10} \subset b_2 \quad \text{Equation 16}$$

$$m_{11} \subset b_0 \quad \text{Equation 17}$$

Let $g_1: M \rightarrow B$ be a function that assigns every minset to a geologic unit that contains the minset. Table 1 gives the function for g_1 the geologic structure shown in Figure 2. Further, for a point $P (x, y, z)$ in a space Ω, a minset $m_{d1 d2 \ldots dq}$ can be assigned a value of $d_k = 0$ or $d_k = 1$ depending on whether $P (x, y, z)$ lies S_k^+ or S_k^-, respectively. This correspondence between every point in the space Ω and minset is expressed by a function $g_2: \Omega \rightarrow M$. Therefore, the function $g: \Omega \rightarrow B$ is expressed to compound the function g_1 and function g_2 as follows:

$$g (x, y, z) = g_1 (g_2 (x, y, z)) \quad \text{Equation 18}$$

This function g that assigns a unique geologic unit to every point in the space Ω is termed the geologic function (Masumoto et al., 1997). It shows that the five point P_1, …, P_5 corresponds to value of g (Figure 3).

3. Generalized Geologic Function

$V\varepsilon$ is a sphere whose radius is ε and its center point P exists inside of the space Ω. When point P exists in geologic unit b, sphere $V\varepsilon$ is also included in geologic unit b if it is possible to make radius ε reasonably short. All the value led geologic functions g of point P in $V\varepsilon$ comprises geologic unit b. However, when point P exists on the boundary surface between lower geologic unit b and upper geologic unit b', two kinds of geologic units b and b' exist in $V\varepsilon$ which means that the value led geologic function g can not be invariable no matter how short radius ε may be. Using this property, we propose a method to distinguish whether the position of point P is in the geologic unit or on the boundary surface.

Table 1:The value of function g_1

minset	g_1 (m)
m_{00}	b_1
m_{01}	b_0
m_{10}	b_2
m_{11}	b_0

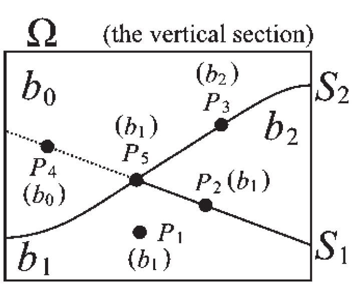

Figure 3: The value of geologic function g on their five points

3.1 Right Above Minset and Right Below Minset

When minset m_0 and m_1 are generated by the surface S which divides space Ω into two subspaces, point P (x, y, z) on surface S is included in m_0 by definition (Figure 4). Point P' $(x, y, z-\varepsilon)$ lies lower than point P (x, y, z) by infinitesimal distance ε (>0) is included in m_0 as well. The minset including point that lies lower than point P (x, y, z) by an infinitesimal distance is termed *the right below minset of point P*. Point P'' $(x, y, z+\varepsilon)$ that lies above point P (x, y, z) by an infinitesimal distance ε is included in m_1. The minset that include point that lie above point P (x, y, z) by an infinitesimal distance is termed *the right above minset of point P*. It can be generally explained by minset that are generated by the surfaces S_1, …, S_q. divides the space Ω into two subspaces. When the concrete properties of surfaces are shown, character string c_1, … c_q is defined by:

$$ck = \begin{cases} 0\ ;\ \text{Point } P \text{ lies lower than } S_k \\ *\ ;\ \text{Point } P \text{ lies on } S_k \\ 1\ ;\ \text{Point } P \text{ lies higher than } S_k \end{cases}$$

$(k = 1, \dots, q)$

Equation 19

When point P does not exist on surface S, character string is composed of binary number, 0 and 1. When point P exists on surface S, character string include *. For example, point P exists on the k th surface S_k but not on other surfaces, the k th character string c_k in the character string c_1 … c_q becomes *, and the other characters becomes either 0 or 1. When it is considered that point P' $(x, y, z-\varepsilon)$ lies lower than point P in an infinitesimal distance ε, it should not be c_k = * but c_k = 0. Hence, character string can be composed by only 0 and 1 when * is replaced with 0. If it is comprehended as a binary number, character string adequately represent subscripts of minset including point P from the definition of minset and point P' $(x, y, z-\varepsilon)$ is also included in the minset. Such minset is termed *the right below minset of point P*. On the other hand, when it is considered that point P' $(x, y, z+\varepsilon)$ lies upper than point P in an infinitesimal dis-tance ε, it should not be c_k = * but c_k = 1. Hence, character string can be composed by only 0 and 1 when * is replaced with 1. Point P is not included in the minset which represents binary subscripts corresponds to the character string, while point P'' $(x, y, z+\varepsilon)$ is include in the minset. Such minset is termed *the right above minset of point P*.

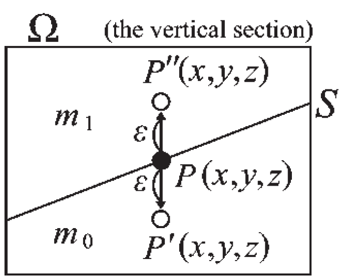

Figure 4: The right below and right above minset of a point P

When point P exists on plural surfaces, it is included plural * in the character string. Thus, we can introduce the concept of right above and right below minset of point P. It is defined generally as follows:

Let character string be c_1 … c_q. When all * are replaced with 0, it is considered that the character string is binary number. The minset which represents binary subscript corresponds to that character string is termed *the right below minset of point P*. When all * are replaced with 1, it is considered that the character string is binary number. The minset which represents binary subscript corresponds to that character string is termed *the right above minset of point P*.

Therefore, there is defined a function $g_2' : \Omega \rightarrow M \times M$ that assigns a pair of the directly above and below minsets (m, m') to every point P (x, y, z) in the objective 3-D space Ω. Next, we explain an example of execution to obtain a pair of directly above and below minsets (m, m')

It can be shown that the five points P_1, …, P_5 correspond to values of g_2'. In the case of point P_1 (x_1, y_1, z_1) that lies lower than both surfaces S_1 and S_2, c_1c_2 is 00:

$$g_2'(x_1, y_1, z_1) = (m_{00}, m_{00})$$ Equation 20

In the case of a point P_2 (x_2, y_2, z_2) on surface S_1 and lower than surface S_2, c_1c_2 is *0. When * interchanges 0 and 1, it gets two character strings 00 and 10:

$$g_2'(x_2, y_2, z_2) = (m_{00}, m_{10})$$ Equation 21

In the case of a point P_3 (x_3, y_3, z_3) that lies higher than surface S_1 and on surface S_2, c_1c_2 is 1*. When * interchanges 0 and 1, it gets two character strings 10 and 11:

$$g_2'(x_3, y_3, z_3) = (m_{10}, m_{11})$$ Equation 22

In the case of a point $P_4(x_4, y_4, z_4)$ on surface S_1 and that lies higher than surface S_2, c_1c_2 is *1. When * interchanges 0 and 1, it gets two character strings 01 and 11:

$$g_2'(x_4, y_4, z_4) = (m_{01}, m_{11})$$ Equation 23

In the case of a point P_5 (x_5, y_5, z_5) on surface S_1 and surface S_2, c_1c_2 is **. When * interchanges 0 and 1, it gets two character strings 00 and 11:

$$g_2'(x_5, y_5, z_5) = (m_{01}, m_{11})$$ Equation 24

3.2 Generalized Geologic Function

Every point P in the space Ω corresponds to a pair of minset (m, m') by function $g_2' : \Omega \rightarrow M \times M$. Therefore, minset m lies right below point P corresponds to geologic unit $g_1(m)$ including the point by function $g_1 : M \rightarrow B$. Such geologic unit is termed *"the right below geologic unit of point P"*. Minset m' lies right above point P is corresponding to geologic unit $g_1(m')$ including the point by function g_1 as well. Such a geologic unit is termed *"the right above geologic unit of point P"*.

Through this method, it is possible to define the function $g' : \Omega \rightarrow B \times B$ that corresponds to a pair of geologic units that lie in both right above and below the point P. The function g' is termed *the generalized geologic function*. When a pair of minset (m, m') corresponds to a pair of geologic units $(g_1(m), g_1(m'))$ by function $g_1' : M \times M \rightarrow B \times B$, the generalized geologic function g' is possible to define by:

$$g'(x, y, z) = g_1'(g_2'(x, y, z))$$ Equation 25

It can be shows that the five points $(P_1, \ldots, P_5)$ of Figure 3 corresponds to value of g' in Figure 5. In the case of a point P_1 (x_1, y_1, z_1) that lies lower than both surfaces S_1 and S_2:

$$g_2'(x_1, y_1, z_1) = (m_{00}, m_{00})$$ Equation 26

$$g_1(m_{00}) = b_1$$ Equation 27

Therefore,

$$\begin{aligned} g'(x_1, y_1, z_1) &= g_1'(g_2'(x_1, y_1, z_1)) \\ &= (g_1(m_{00}), g_1(m_{00})) \\ &= (b_1, b_1) \end{aligned}$$ Equation 28

In the case of a point P_2 (x_2, y_2, z_2), P_3 (x_3, y_3, z_3), P_4 (x_4, y_4, z_4) and P_5 (x_5, y_5, z_5):

$$g'(x_2, y_2, z_2) = (b_1, b_2)$$ Equation 29

$$g'(x_3, y_3, z_3) = (b_2, b_0)$$ Equation 30

$$g'(x_4, y_4, z_4) = (b_0, b_0)$$ Equation 31

$$g'(x_5, y_5, z_5) = (b_1, b_0)$$ Equation 32

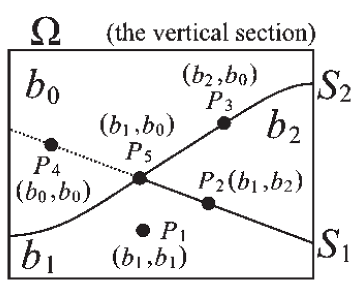

Figure 5: The value of generalized geologic function g' on their five points

3.3 Determiner Algorithm of Minset Right Below and Right Above the Optional Point

The processing to determine the character string and the processing to substitute * for 1 or 0 can be done all together as follows, yet it is necessary to let the elevation of surface S_1, …, S_q dividing space Ω into two, upper and lower be $z_k = s_k(x, y)$ $(k = 1, \ldots, q)$ and let the difference between the elevation H of point P and z_k be $D_k = H - z_k$.

1. d_k $(k = 1, \ldots, q)$ are given by:

$$\left.\begin{array}{l} D_k > 0 \,;\, d_k = 1 \\ D_k = 0 \,;\, d_k = 0 \\ D_k = 0 \,;\, d_k = 0 \end{array}\right\} \qquad \text{Equation 33}$$

The q digit binary number $d_1'd_2'...d_q'$ represent the index of minset right below point P.

2. d_k' $(k = 1, \ldots, q)$ are given by:

$$\left.\begin{array}{l} D_k > 0 \,;\, d_k' = 1 \\ D_k = 0 \,;\, d_k' = 1 \\ D_k < 0 \,;\, d_k' = 0 \end{array}\right\} \qquad \text{Equation 34}$$

The q digit binary number $d_1'd_2'...d_q'$ represent the index of minset right above point P.

Minset right below and right above point P can be determined by Equations 33, 34, yet it is necessary to pay attention, when we introduce this method into computer processing. Numerical calculation by the computer is approximate calculation of finite numbers of digit. Since it should not be $D_k = 0$ when we deal with difference D_k between point P and surface S_k as real number. Point P should be a point on the surface when point P (x, y, z) lies within the elevation z_k of surface in the range $\pm\,\varepsilon$ (Figure 6). In this case, Equations 33, 34 become as follows:

1'. d_k' $(k = 1, \ldots, q)$ are given by:

$$\left.\begin{array}{l} D_k > \varepsilon \,;\, d_k = 1 \\ |D_k| \quad \varepsilon \,;\, d_k = 0 \\ D_k < -\varepsilon \,;\, d_k = 0 \end{array}\right\} \qquad \text{Equation 33}'$$

The q digit binary number $d_1d_2...d_q$ represent the index of minset right below point P.

2'. d_k' $(k = 1, \ldots, q)$ are given by:

$$\left.\begin{array}{l} D_k > \varepsilon \,;\, d_k'=1 \\ |D_k| \quad \varepsilon \,;\, d_k' = 1 \\ D_k < -\varepsilon \,;\, d_k' = 0 \end{array}\right\} \qquad \text{Equation 34}'$$

The q digit binary number $d_1d_2...d_q$ represent the index of minset right above point P.

4. Geologic Modeling and Visualization of Geologic Boundaries on Geomodel2003

We added algorithms to distinguish the geologic boundary using the generalized geologic function and produced Visual Basic program Geomodel2003. Examples of the various geologic maps are presented in Figure 7 using Geomodel2003. The study area (8.7 × 6.5 km) is located in the Honjyo region of Akita Prefecture, Northeast Japan using data extracted from a geologic map (Osawa et al., 1977). The mapped district is underlain by Neogene rocks and Quaternary alluvium. The Neogene formations, 3,000 m to 5,000 m in total thickness, consist mainly of sedimentary rocks with acid tuff. The main part of this area is characterized by intense folds with a general trend in the N-S direction. The Quaternary alluvial deposits unconformably overlay the Neogene formations, and are widely distributed along rivers

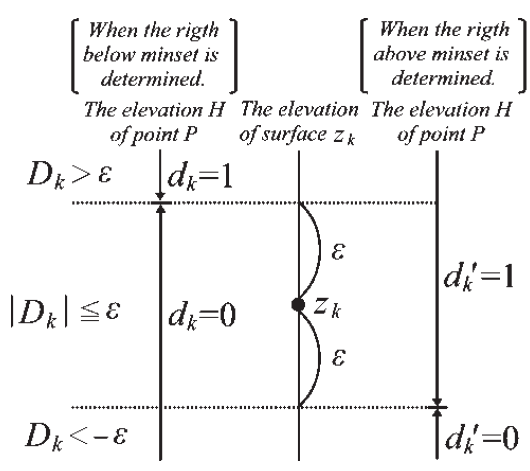

Figure 6: Determination of minset right below and right above the optional point

sections. The distribution of geologic units is shown in Figure 7(a) as the 3-D geologic map. Figure 7(b) is the 3-D geologic map in the case of including the geologic boundaries. Figure 7 (c) is the 2-D geologic map. Figure 7(d) is the vertical geologic section map on the line (A)–(B) in Figure 7(c).

5. Conclusion

We defined the generalized geologic function which corresponds to a pair of geologic units laying both right above and right below the every point in the objective 3-D space based on the geologic function. It is possible to distinguish the geologic boundary by applying the generalized geologic function to points on surfaces. We developed the theory and algorithms to visualize the geologic boundary, and through this concept produced Visual Basic program Geomodel2003. As we improved the sub-routine which had been developed before, the geologic boundaries can resultantly be visualized on various geologic maps. There should be considerable potential to represent geologic boundaries in the GRASS GIS, when this method is introduced to GRASS GIS.

Acknowledgment

This research was partially supported by the Japanese Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Scientific Research (C) (2), Project No. 14540430.

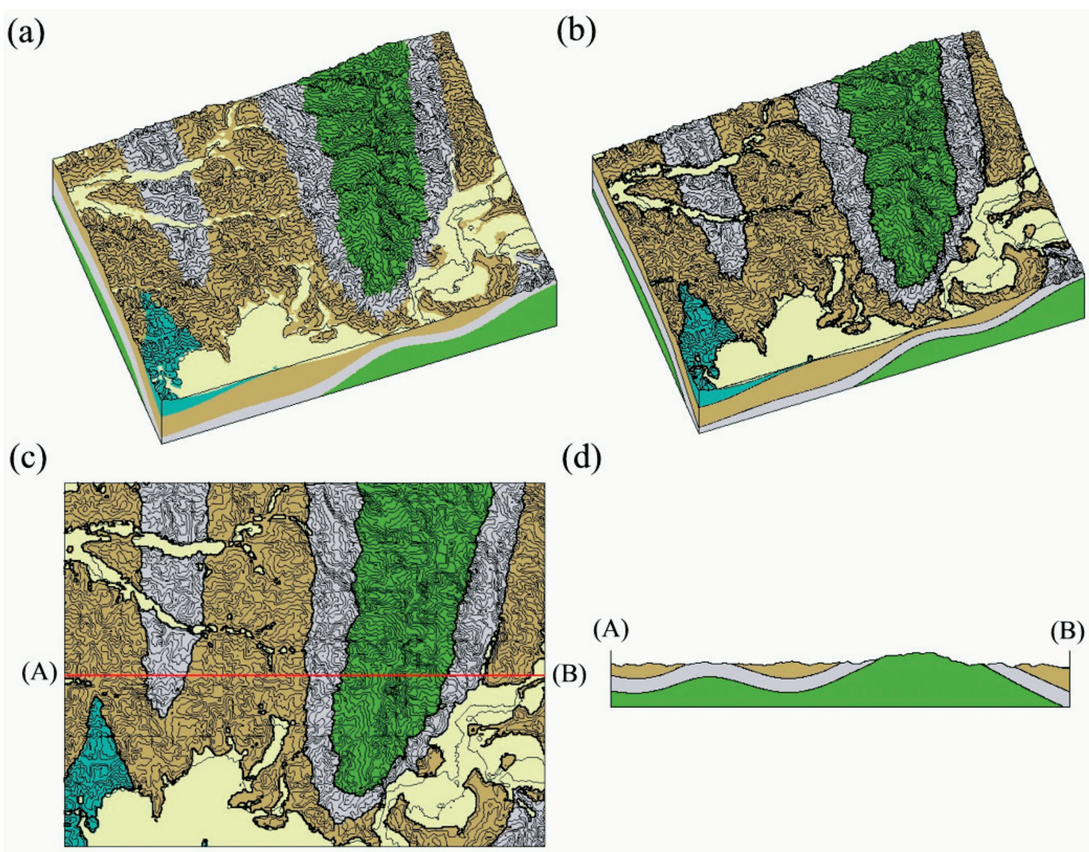

Figure 7: Example of the various geologic maps using Geomodel2003, (a), (b) the 3-D geologic map (in the case of including the geologic boundaries and not.), (c) the 2-D geologic map, (d) the vertical geologic section map on (c)

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