Local Geoid Modeling for Thailand

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Abstract
This paper aims to present the computations of two geoid models for Thailand. The first model, THAI12G, is a gravimetric geoid, referenced to the geocentric WGS84 ellipsoid, computed through one-dimensional spherical Fast Fourier Transform. The other model is THAI12H, which is a hybrid geoid that encompasses all gravimetric information of THAI12G as well as the 200 GPS ellipsoid heights (in the national WGS84 geodetic datum) co-located with orthometric heights (in the national Kolak vertical datum of 1915 (Kolak-1915)) through least-squares collocation (LSC). The non-tidal EGM2008 global geopotential model from degree 2 to 2190 and 3,949 terrestrial gravity measurements were used to contribute long- and medium-scale information of geoid structure. In the mountainous terrains devoid of gravities, the topography-implied gravity anomalies were simulated using the high-resolution residual terrain model (RTM) data from a three-arcsecond digital elevation model. Fits of 200 GPS/leveling reference points to THAI12G showed a 60.6-cm root mean square (rms) with an estimated offset of +71.5 cm around a 0.126-ppm north-south tilted plane. After applying LSC conversion surface to finally obtain THAI12H, the rms of the fit between the model and the same reference points reduced to 5.7 cm (no tilts and zero average).The THAI12H model was assessed using 53 GPS/leveling check points, yielding an overall rms of 16.1-cm.

1. Introduction

Geoid is an equipotential surface of the Earth's gravity field that best fits to global mean sea level in a least squares sense (Jekeli et al., 2009). With the aid of the Global Positioning System (GPS), height-system modernization is based on a fundamental equation that connects GPS-derived heights, h, above World Geodetic System 1984 (WGS84) ellipsoid, and orthometric heights, H, referred to a national vertical datum (i.e., $H = h - N$, where N is the (local) geoid undulation with respect to the ellipsoid). However, to obtain an orthometric height, it depends on an accurate geoid undulation that is determined from the required accuracy and resolution of gravimetric data on or near the Earth's surface. By these requirements, the determination of a geoid model for Thailand was difficult in previous years, due mainly to the insufficient coverage and distribution of gravimetric data in the country as well as no possibility to access the data in neighboring countries. In recent years, the number of the gravity points measured by Royal Thai Survey Department (RTSD) has been increased, and sufficient to be used for constructing a local geoid model for Thailand. The intention has substantially grown after the National Geospatial-intelligence Agency (NGA) officially released the latest Earth Gravitational Model of 2008 (EGM2008) in 2008 (Pavlis et al., 2012).

This study presents the first attempt to geoid determination in Thailand for supporting the conversion between the GPS ellipsoid heights (h) in the national WGS84 geodetic datum and the orthometric heights (H) referred to Kolak-1915. In Thailand, most of gravity points were measured along with existing roads and accessible areas, which left the mountainous areas devoid of gravities. These data gaps could produce the significant errors resulting from data interpolation on a specified grid for geoid computation. Therefore, we filled in the void areas with topography-implied gravity anomalies (Hirt et al., 2010) by utilizing high-resolution residual terrain model (RTM) data (Forsberg and Tscherning, 1981 and Forsberg, 1984) and EGM2008. The RTM data was constructed from a three-arcsecond digital elevation model (DEM) [e.g. the Shuttle Radar

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Topography Mission (SRTM) (Javis et al., 2004 and Rodriguez et al., 2005): version 4 (void-filled areas) available at http://srtm.csi.cgiar.org/] that contributed high-frequency gravity field signals. EGM2008-only gravity anomalies were used to pad coastal and marine areas as well as neighboring countries to reduce spurious features during gridding of the areas (Claessens et al., 2011).

The THAI12G gravimetric, geocentric geoid model was computed using the combination of EGM2008, RTSD terrestrial gravity anomalies, and topography-implied anomalies through remove-and-restore technique (Sansó and Rummel, 1997). The computation area spans from 5°N to 25°N and 95°E to 110°E. The SRTM digital elevation model was used to generate terrain corrections for the computation of Faye anomalies (free-air anomalies plus terrain corrections). The method for computing geoid undulations was based on the use of the well-known one-dimensional (1-D) spherical Fast Fouier Transform (FFT) of Haagmans et al., (1993) to evaluate Stokes' integral (e.g. Yun, 1999, Smith and Milbert, 1999, Featherstone et al., 2001 and Smith and Roman, 2001).

The least-squares collocation is a useful and powerful method to apply for a variety of problems in interpolation and prediction (Moritz, 1980, You, 2006 and You and Hwang, 2006). It is a useful tool for the combined a gravimetric model with GPS/leveling points through a conversion surface (Smith and Milbert, 1999 and Smith and Roman, 2001). Such an approach was adopted for Thailand, leading to the THAI12H hybrid geoid model that directly related GPS heights in WGS84 datum to orthometric heights in Kolak-1915. In fact, although the surface fitting by least-squares collocation gives a practical useful product for a more direct transformation of GPS heights to orthometric heights, it does not necessarily provide an improved model of gravimetric geoid (Featherstone et al. 2010). It does not mean that the problems with a vertical datum (e.g. Kolak-1915) have been resolved. Moreover, the gravimetric geoid model is not being corrected, rather it is distorted to fit the vertical datum that also contain errors.

In this paper, the entire geoid computation process was discussed. The comparative evaluation between THAI12H geoid undulations and 53 GPS/leveling check points was made, and its numerical result was also discussed. Finally, the conclusions were summarized.

2. Data Preparations

2.1 The Earth Gravitational Model of 2008, EGM2008

EGM2008 is the latest version of geopotential model, following EGM96 (Lemoine et al., 1998) that utilizes world gravity data at a 30 arcminute resolution. The model is more ambitious, and has maximum resolution of 5 arcminute, based on improved long wavelength information from GRACE, improved terrain data and altimetry data, and reliable and updated surface gravity database (Kenyon et al., 2007). EGM2008 is complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159. In the tests with geoid undulations derived from 200 GPS/leveling (ITRF2005/Kolak-1915) reference points in Thailand, the undulations were compared to those from EGM96 and EGM2008. The global geoid undulations were generated using the public domain software f477.f for EGM96 and hsynth_WGS84.f for EGM2008 (more details available at http://earth-info.nga.mil/GandG/wgs84/gravitymod/). The statistic differences, summarized in Table 1, show that the standard deviation of EGM2008 (±15.2 cm) is smaller than that of EGM96 (±27.2 cm). EGM2008 showed an obvious improvement and better resolved smaller scale features. Hence, it was chosen as the reference model in all computations, providing its structures of long (and feasibly some medium) wavelengths. The range of spherical harmonic coefficients used is degree and order 2 to 2190, which corresponds to a minimum spatial resolution of about 5 km.

Table 1: Comparison of global geopotential and GPS/leveling geoid undulations (units in m)

200 stations	min	max	mean	std
EGM96	–0.804	+1.250	+0.530	±0.272
EGM2008	+0.086	+1.209	+0.587	±0.152

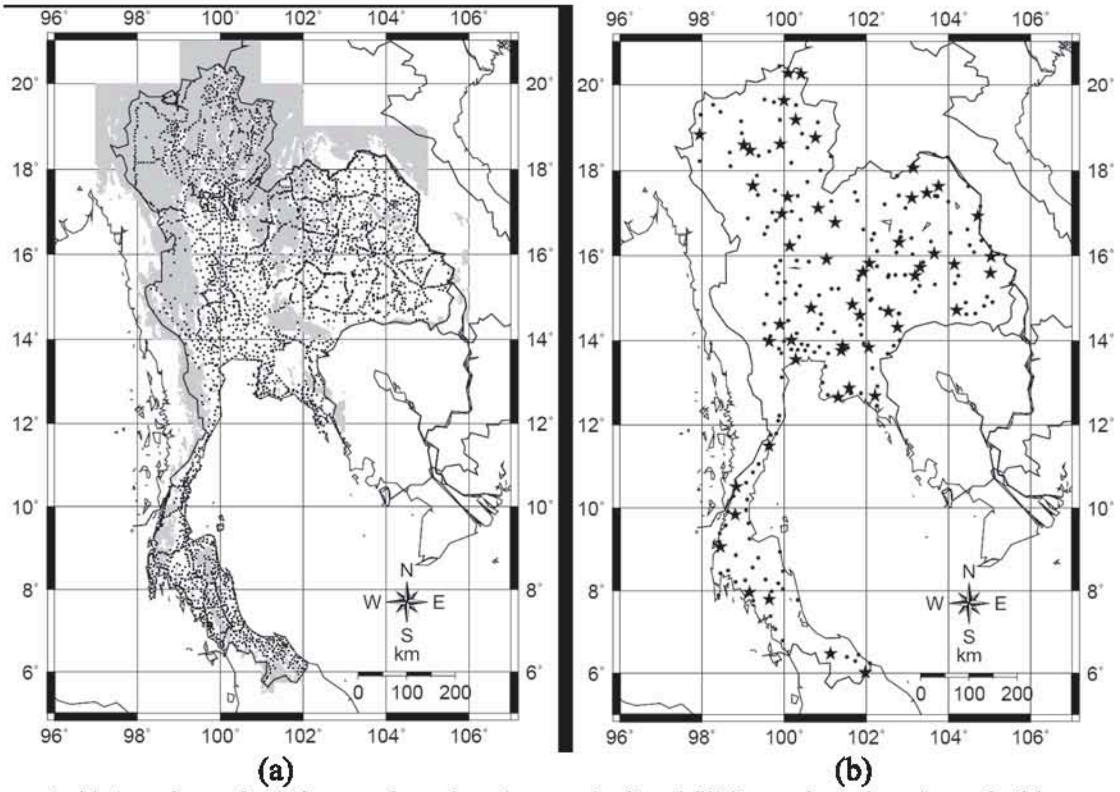

Figure 1: (a) Locations of existing gravity values (square dots) and fill-in gravity values (gray shade); (b) Locations of 200 GPS/leveling reference points (dots) and 53 GPS/leveling check points (star dots)

2.2 Terrestrial Gravity Anomalies

Figure 1a depicts 3,979 terrestrial gravity stations (square dots) in the boundary of Thailand, provided by RTSD (RTSD, 2007). Some parts of the region, e.g. north and west areas, are mountainous and inaccessible. Obviously, the resolutions in those areas are not uniform, with data mostly following existing roads, and large data gaps are in the order of over 50 km (27.8 arcminute); these may produce systematic effects in geoid computation. In the other areas, the distributions of data are more uniform, and their resolutions vary from 2 to 10km (1.1 to 5.5 arcminute). The gravity network was referred to the International Gravity Standardization Net 1971 (IGSN71). RTSD gravity stations were checked for outliers. Only 3,949 stations were used for geoid computations as the other 30 stations remained questionable, and thus they were excluded. The locations and heights of the stations were coarsely measured, and also the adjustment of gravity networks was not made clear. As such, the gravity values may not represent the actual gravity field over areas of interest (Abeyratne et al., 2009), and can postulate errors to the computation of geoid undulations. Such commission errors can be reduced only with improvements in the data, namely by reducing observational errors (Jekeli et al., 2009). However, for the first time of our experimental geoid modeling, they (the errors) are not considered in this present work. For anomaly computations, all gravity anomalies were recalculated with respect to WGS84 (G1150) reference ellipsoid to be compatible with WGS84 datum.

2.3 Fill-in Gravity Anomalies

As shown in Figure 1a, there are coarse gravity networks in northern and western parts of Thailand. For accurate geoid determination, refine-resolution gravity points are required. The approach developed by Hirt et al., (2010) is applied in order to fill topography–implied gravity anomalies in the void areas where gravity points are not available (see color shade in Figure 1a). It is based on the residual terrain model approach after Forsberg and Tscherning (1981). The idea of it is that medium-elevated and rugged terrain can be modeled by RTM data which represent high-frequency gravity signals. RTM modeling uses the three-arcsecond SRTM digital elevations to represent Earth's topography. The (five-arcminute) DTM2006.0 spherical harmonic model of Earth's topography (Pavlis et al., 2012) is used as a long-wavelength reference surface to remove low-frequency components. The RTM terrain effect under planar approximation (Sansó and Rummel, 1997 and Bajracharya, 2003) is given by

$$\delta g_{RTM} \approx 2\pi k\rho(h - h_{ref}) - C$$

Equation 1

where k is Newton's gravitational constant, ρ is an average density of the topographic mass (the crust). The C terrain correction can be computed according to Forsberg (1984) using SRTM rectangular prisms. The symbols, h_{ref} and h, represent the heights of the reference surface and the topographic surface, respectively.

In this study, we require gravity anomalies in rugged terrains (where no measured gravities on land are available) to diminish interpolation errors in geoid computation. We assume that, in the void areas, EGM2008 contributes long and medium-wavelength information of the earth's gravity field. For all wavelength contents, the topography-implied gravity anomalies in those areas can be approximated by EGM2008 gravity anomalies, Δg_M, and δg_{RTM} as follows:

$$\Delta g \approx \Delta g_M - \delta g_{RTM} - 2\pi k\rho h_{ref}$$

Equation 2

In Equation 2, refined Bouguer gravity anomalies [complete Bouguer gravity anomalies plus terrain corrections (Heiskanen and Moritz, 1967)] are immediately obvious if we consider Δg_M as free-air gravity anomalies.

We consider filling the topography-implied (or simulated) anomalies in the areas with higher 400-m elevations because of not only less correlation (linear relationship) of RTSD free-air anomalies with respect to lower elevations (not shown in this work) but also the number of data and the distribution of these data, similar to the case study of geoid computation in the Malaysian peninsula as stated in Vella (2003). However, lower elevations could be significant, but are not considered in this work. The simulated anomalies were derived from three-arcsecond SRTM data (averages over 30 arcsecond × 30arcsecond blocks). The augmentation of the existing gravity data with the simulated data in color shade is shown in Figure 1a. The inclusion of the simulated data aids to control data gridding before the step of FFT geoid computations that requires gravity data on regular grids. We use EGM2008-only to mitigate the edge effects in the geoid computation due to no gravity data available in ocean areas and land areas outside the Thailand territory. A topography-implied anomaly was interpolated to each RTSD point using bivariate interpolation (Akima 1974 and 1978) for data comparisons. Table 2 provides statistics for gravity anomalies and corresponding comparisons, for instance, in the mountainous area bounded by latitude of 18°N - 20°N and longitude of 97°E - 102°E, thus also demonstrating the improvement (and also consistency) in the modeling relative to the existing gravity data. The standard deviation of the difference of the topography-implied anomalies and RTSD refined Bouguer gravity anomalies decreases to ±18.305 mGal.

Table 2: Statistics of gravity anomalies and data augmentation (units in mGal)

No	Quantity	No. of points	min	max	mean	std
1	RTSD refined Bouguer anomalies	3,949	−159.051	+122.597	−23.097	±22.851
2	EGM2008 at RTSD stations	3,949	−75.717	+70.945	−13.581	±19.719
3	RTSD refined Bouguer anomalies: 18°≤φ≤20° and 98°≤λ≤102°	471	−159.051	+122.597	−46.042	±22.548
4	EGM2008 at RTSD stations: 18°≤φ≤20° and 98°≤λ≤102°	471	−75.717	+70.945	−19.925	±23.890
5	Data padding in the higher-400m-elevation areas at RTSD stations: 18°≤φ≤20° and 98°≤λ≤102°	471	−124.434	+95.884	−8.956	±27.586
6	3 – 4	471	−126.892	+106.314	−26.117	±21.208
7	3 – 5	471	−134.468	+53.020	−37.657	±18.305

2.4 GPS/Leveling Data

In 2002, the RTSD completed the adjustment of national geodetic network in WGS84 (geocentric) datum (RTSD 2003) in accordance with the standard of Federal Geodetic Control Committee (FGCC) (Bossler, 1984). The RTSD networks were categorized into three levels as follows: (1) reference frame, (2) primary network, and (3) secondary network. The reference frame (zero order network) consists of 7 GPS stations established every part of Thailand. In 2008, the (zero) network was recomputed to map ITRF2005 after the concurrence of the 9.2 Mw Sumatra-Andaman earthquake on the 26th December of 2004; the previous realizations of the network were tied to ITRF94, ITRF96, and ITRF2000 (Satirapod et al., 2009). There are 18 GPS stations in the primary (first order) network with the interval of about 250km for each station. This network was extended from the zero order network. For secondary (second order) network, 692 GPS stations were extended from the primary stations. The station spacing ranges from 20 to 50km, and its accuracy is around 1 ppm. However, the geodetic coordinates, particularly ellipsoidal heights, were referred to WGS84 (G1150) ellipsoid, which has not yet been linked to any ITRF's. Therefore, we assume that WGS84 (G1150) aligns to ITRF2005, though this could be a cm-level error source (NIMA, 1997 and Wilney, 2009) in geoid computation.

For a number of years, Kolak-1915 vertical datum has still remained the official vertical datum in Thailand. The origin of it was realized based on tidal observations carried out between 1910 and 1915 at Kolak island using one tide-gauge station located at latitude 11°47′42″N and longitude 99°48′58″E. For vertical control network of the first order leveling, 333 primary benchmarks with orthometric heights were extended from the origin point to every part of the country. More than 1,600 secondary benchmarks were tied to the primary control network. However, because the shape of the country looked like an ancient axe or a long trunk, the adjustment of the primary network was separately conducted in two areas [upper and lower areas at the origin point (latitude: 11°47′42″N)] by minimally constrained adjustment (fixed to just the origin point). This may cause inconsistencies in the vertical datum over the region besides gross (undetected mistakes) and systematic errors in spirit-leveling. In this study, only 253 leveling stations co-located with the GPS heights on the horizontal network stations were available. Figure 1b shows the distribution of 200 GPS/leveling reference stations (dots) and 53 GPS/leveling check points (star dots). These stations are rather patchy, and their spacing is variable, ranging from 25 to 100-km spacing. The irregular distribution of these stations occurs in rugged terrains, especially, in the north-western part of the country. The quality of the leveling stations is ambiguous and difficult to identify because they come from different orders of spirit-leveling, i.e. first, second, and third orders. It must be emphasized that the accuracy of these heights may not be equally accurate as one would expect, but the spirit-leveling should not exceed 12mm-square-root-km allowable misclosure (the third order). For this reason, the leveling heights may not be such a strong validation of the geoid models (i.e. THAI12G and THA12H).

3. Computation of THAI12G

In this work, the geoid undulation of the THAI12G gravimetric (and geocentric) geoid model, N, is computed through the generalized Stokes' integral (Heiskanen and Moritz, 1967). All computations are in the non-tidal system. With the usual remove-and-restore procedure, the geoid undulation is defined as follows:

$$N = N_M + \frac{R}{4\pi\gamma} \iint_{\sigma} (\Delta g_F - \Delta g_M) S(\psi) d\sigma + \delta N_I$$

Equation 3

where σ is the area of integration, R is the mean radius of the Earth, γ is normal gravity on WGS84 ellipsoid (Somigliana's formula in Heiskanen and Moritz (1967), $S(.)$ is Stokes' function with spherical distance ψ, and Δg_{Fa} is the free-air gravity anomaly with terrain correction (called Faye anomaly , used to approximate Helmert gravity anomaly). The symbols "Δg_M" and "N_M" are the gravity anomaly and the geoid undulation, generated by EGM2008 at degree 2 to 2190, respectively; more details can be found in Pavlis et al., (2012). In most of geoid computations, changes in topographic masses result in the change in the geopotential of geoid. Such a systematic change is called the indirect effect, δN_I, and is given by (Wichienchareon, 1982):

$$\delta N_I = -\frac{\pi k \rho H^2}{\gamma}$$

Equation 4

Terrain corrections were applied to all (RTSD) measured gravity points. The numerical integration of the corrections was performed using the analytical formula for the gravitational effect of a homogeneous rectangular prism (Forsberg, 1984). We use three-arcsecond SRTM data, corresponding to 90m × 90m prisms with an average topographic mass (crust) density of 2670 kg/m^3, for producing terrain corrections. Then, the topography-implied anomalies in Equation 2 were augmented to the RTSD data (before gridding). One dimensional (1-D) spherical Fast Fourier Transform (FFT) of Haagmans et al., (1993) was used to evaluate Stokes' integral in Equation 3. The FFT requires gridded data. Thus, the grid of refined (terrain-corrected) Bouguer anomalies was interpolated from the scatteredly measured points using a method of continuous curvature spines in tension in the Generic Mapping Tools (GMT) (Smith and Wessel, 1990 and Wessel 2009). The tension factor of T = 0.75 was selected to minimize the impact of gravity errors in mountainous areas on adjacent grid points without gravity data as suggested by Smith and Milbert (1999). To reduce spatial aliasing effects in the presence of high-frequency signal, mean gravity anomalies were constructed using reconstructing technique describing in GMT. The mean anomalies were interpolated to 30 arcsecond grid. Then, we restored Bouguer plates to the anomalies using 30 arcsecond elevation data, derived from three-arcsecond SRTM data (averages over 30 arcsecond × 30 arcsecond blocks), yielding a grid of Faye anomalies. The residual co-geoid undulations on a regular grid with a spacing of 30 arcsecond were computed, according to the second term of Equation 3 using 1-D spherical FFT in the area defined by 5° ≤ ϕ ≤ 25° in latitude and 95° ≤ λ ≤ 110° in longitude. The FFT was conducted on the residual grid, $\Delta g_F - \Delta g_M$ using 100% zero-padding on the east and west edges of the grid to eliminate the effect of cyclic convolution. The geoid undulations of THAI12G were obtained by the restoration of N_M and the inclusion of δN_I. Those values of δN_I were generated using 30 arcsecond mean elevations for H in Equation 4. The values of THAI12G (gravimetric and geocentric) geoid undulations vary from about –5 m in the southernmost area to –40 m in the northernmost area of the country (its figure is not shown in this paper).

For evaluation, the geoid undulations of THAI12G were compared with the undulations, N_{Kolak}, derived by 200 GPS/leveling reference points (see Figure 1b). Figure 2 shows the plot of the differences of N_{Kolak} and N, and their statistics are listed in Table 3. The values of difference range from +8.7 to +119.6 cm with a standard deviation of ±15.1 cm, and have an average bias of +58.6 cm, which implies that Kolak-1915 is above THAI12G. As compared to Table 1, THAI12G performs equivalently to EGM2008 (std = ±15.2 cm), i.e. no significant differences between two models. This may indicate that EGM2008 can be used alone over Thailand if EGM2008 always yields heights compatible with (tilted and distorted) Kolak-1915—the similar case can be found in Feathestone et al., (2010) and Clasessens et al., (2011). However, these numerical findings signify that the addition of RTSD gravity data does not deteriorate the long-and-medium wavelength structures of EGM2008 in THAI12G. Also, the accuracy of THAI12G can be significantly improved if more ground gravimetry has been conducted, which are currently under way. Also investigated was the magnitudes of biases for two areas, upper and lower the origin point of Kolak-1915 (ϕ = 11°47′42″N). Table 3 summarizes statistics of the differences. The results show a 66.3-cm bias of the lower area and a 56.9-cm bias of the upper area with standard deviations of ±17.8 cm and ±14.0 cm, respectively, signifying that a tilt exists in Kolak-1915. These disagreements may correspond to datum inconsistencies from error sources of the data used, e.g. leveling heights, gravity data, DEM, and EGM2008 data. Further analyses of the observational errors that can propagate to the computed geoid undulations should be made, but these will not be considered in this present work.

Table 3: Statistics of differences between the geoid undulations implied by GPS/Leveling points and THAI12G (units in m)

area	No. of points	min	max	mean	std	rms
Entire areas	200	+0.087	+1.196	+0.586	±0.151	±0.606
Upper area: ϕ>11°47'42"N	163	+0.087	+0.967	+0.569	±0.140	±0.586
Lower area: ϕ≤11°47'42"N	37	+0.407	+1.196	+0.663	±0.178	±0.686

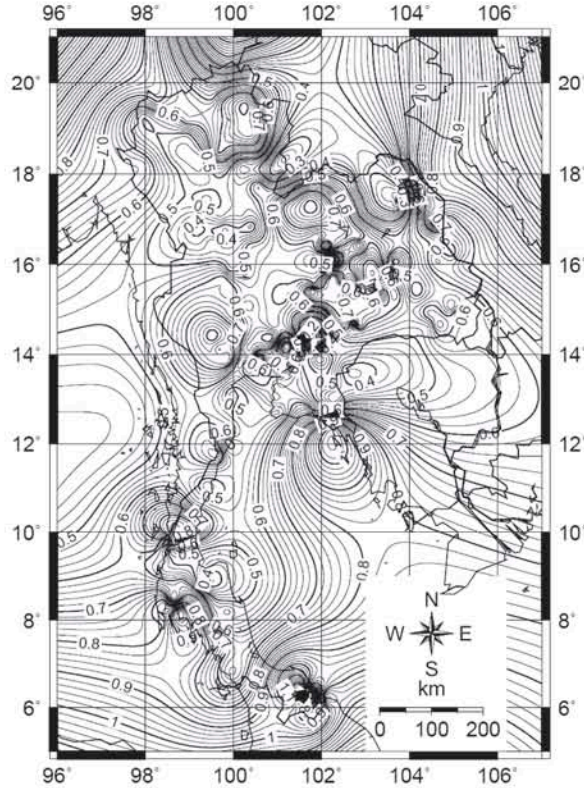

Figure 2: Differences of 200 GPS/leveling derived geoid undulations and THAI12G geoid undulations; contour interval = 0.02 m

4. Computations of the Conversion Surface and THAI12H

Based on the foregoing results, the comparison of a geoid model (i.e. THAI12G) and GPS/leveling data set provides a means of estimating, and removing, possible systematic errors the geoid model, leveling, or GPS measurements (Smith and Milbert, 1999). Least-squares collocation is used for modeling the combined geoid, GPS and datum inconsistency in Kolak-1915 to acquire a conversion surface. Subtracting this surface from the THAI12G model yields the hybrid geoid model, THAI12H. The model of combining THAI12G geoid undulation (N), GPS height (h_{GPS}), and Kolak-1915 orthometric height (H_{Kolak}) errors is defined by forming the residuals, e, in the following,

$$e = (h_{GPS} - H_{Kolak}) - N .$$

Equation 5

The residuals, e, of using 200 points had an average of +58.6 cm with a standard deviation of ±15.1 cm (see Table 3). As shown in Figure 2, it seems clear that there exist datum inconsistencies in the region. Because of the requirement of centered data for least-squares collocation (Moritz, 1980), we modeled a tilt plane by using a simple form of the first order polynomial surface: $f(x,y) = a_0 + a_1x + a_2y$. Such a trend surface was removed from the residuals in Equation 5. The estimated parameters are summarized in Table 4. These significant parameters show that Kolak-1915 may contain medium wavelength errors (You, 2006). The estimated bias of +0.715 m corresponds to the mean offset between Kolak-1915 and THAI12G in a least-squares sense. A significant tilt (–0.126 ppm or mm/km) occurs in north-south direction while an east-west tilt (+0.004 ppm) is much smaller.

By the principle of least-squares collocation, the $\tilde{s}$ vector of predicted (detrended) residuals on a 30 arcsecond × 30 arcsecond grid is calculated using the formula,

$$\tilde{s} = C_{st}[C_{tt} + C_{nn}]^{-1} l$$

Equation 6

where l is the vector of detrended residuals, C_{st} is the covariance matrix between predicted residuals and observations (detrended residuals), and C_{tt} is the covariance matrix between observations. The symbol "C_{nn}" represents the covariance matrix of random errors (or noises) in the residuals. The full matrix of C_{nn} was difficult to obtain because we had a limited knowledge of random errors, for instance, in gravimetric geoid noises, the leveled Kolak-1915 heights, and GPS heights during the time of geoid modeling. Furthermore, most of the information in the region changed between the 1990s and 2000s. For simplicity, we assume no correlation between observations. Thus, C_{nn} is defined by $\sigma_0^2 I$, where σ_0^2 is a priori variance and I is an identity matrix. The covariance matrices, C_{st} and C_{tt}, were derived from a covariance function. It (covariance function) was empirically computed based on the use of detrended geoid residuals, and was fit by the simple form of a Gaussian (exponential) covariance function as follows:

$$C(d) = C_0 \exp(-\frac{d}{L})$$

Equation 7

where d is the distance between points (km), C_0 is a function variance (km^2), and L is a correlation length (km). Figure 3 shows the plot of the empirical covariance function (dots) with the Gaussian function fit of C_0 = 0.016 m^2, and L = 30 km (solid line).

For the estimation of σ_0^2 in $\mathbf{C}_{nn}$, we made a few iterations of the prediction process to assign the value of σ_0^2 consistent with the residual misfit about the predictions of Equation 4 as stated by and Milbert (1997) and Smith and Roman (2001). We found that the rms of residuals from the prediction step matched the assigned noise, when $\sigma_0^2 = (5.5)^2$ cm^2 was used for 200 points (as described below). Then, the predicted residuals, $\tilde{s}$, on a 30 arcsecond grid were computed.

The trend surface was computed on a 30 arcsecond grid using the parameters, listed in Table 4. This surface was restored to the grid of the predicted residuals, $\tilde{\mathbf{s}}$, to provide the conversion surface. Removing the (conversion) surface from THAI12G produces the final hybrid model, THAI12H, which directly connects WGS84 (ITRF2005) ellipsoid heights and Kolak-1915 orthometric heights. Figure 4 shows the conversion surface, which is rather smooth and similar to the residual geoid undulations in Figure 2, but it is not reliable outside the boundaries of Thailand. Finally, THAI12H were compared with 200 GPS/leveling reference points to test whether or not the conversion process was successful. The comparison yielded a 5.7-cm rms of fit, with no offset (zero mean) for entire areas. When the upper and lower areas were considered separately, there were no offsets for both areas, meaning that systematic errors were removed and, thus, the conversion process seems to be successful (cf. $\sigma_0^2 = (5.5)^2$ cm^2 in $\mathbf{C}_{nn}$).

The THAI12H geoid undulations, N, was evaluated by comparing with the GPS/leveling-derived geoid undulations, N_{Kolak}, at 53 check points. Figure 5 shows the locations of 53 GPS/Leveling check points in star dots and the discrepancy distribution between N_{KoLak} and N. EGM96 and EGM2008 were transformed to fit 200 GPS/leveling (ITRF2005/Kolak-1915) reference points by removing their bias and tilts. The statistics of the differences, N_{Kolak} minus N, are summarized in Table 5. The values of statistics show the more improvement of THAI12H than EGM96. However, there is a marginal improvement in the standard deviation of the difference of THAI12H (±15.8 cm) over EGM2008 (±17.0 cm). Large discrepancies in the order of sub-meter appear in the north and Middle East areas of higher 400-m elevations (see also Figure 1a). The overall agreement between N_{KoLak} and N is ±16.1 cm in term of rms.

Table 4: The estimation of trend parameters

Parameter	Estimated value
bias or offset (a_0)	+0.715 m
tilt in East-West direction (a_1)	+0.008 mm/km [ppm]
tilt in North-South direction (a_2)	–0.126 mm/km [ppm]

Table 5: Statistics of differences between geoid undulations implied by 53 GPS/Leveling check points minus EGM2008 and EGM96 with the removal of bias and tilts (units in m)

model	min	max	mean	std	rms
THAI12H	–0.493	+0.539	+0.034	±0.158	±0.161
EGM2008	–0.418	+0.324	+0.002	±0.170	±0.170
EGM96	–0.726	+0.983	+0.023	±0.305	±0.303

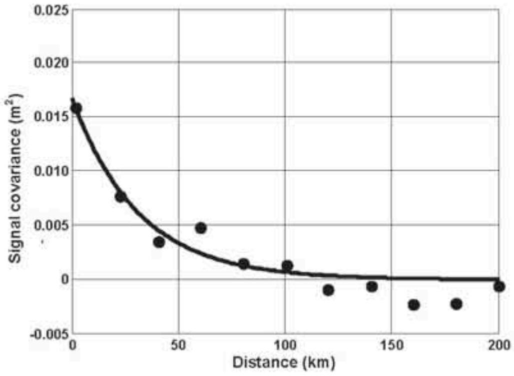

Figure 3: Empirical covariance function (dots) and Gaussian covariance function (solid line)

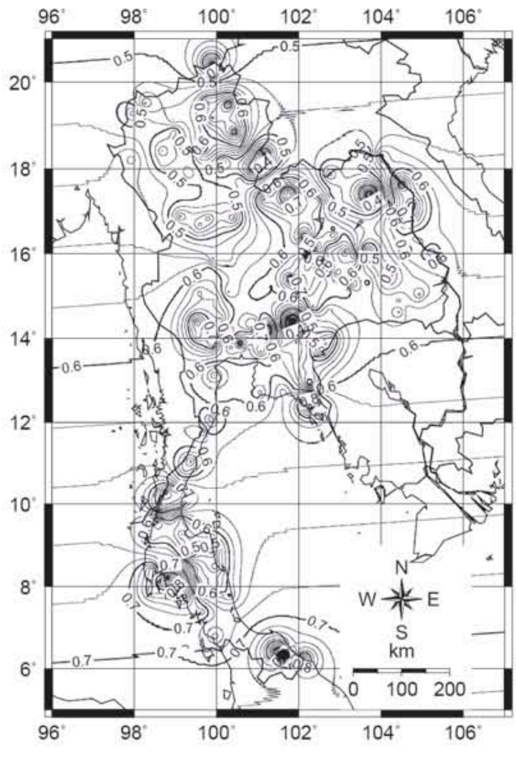

Figure 4: The conversion surface relating THAI12G to THAI12H; contour interval = 0.02 m

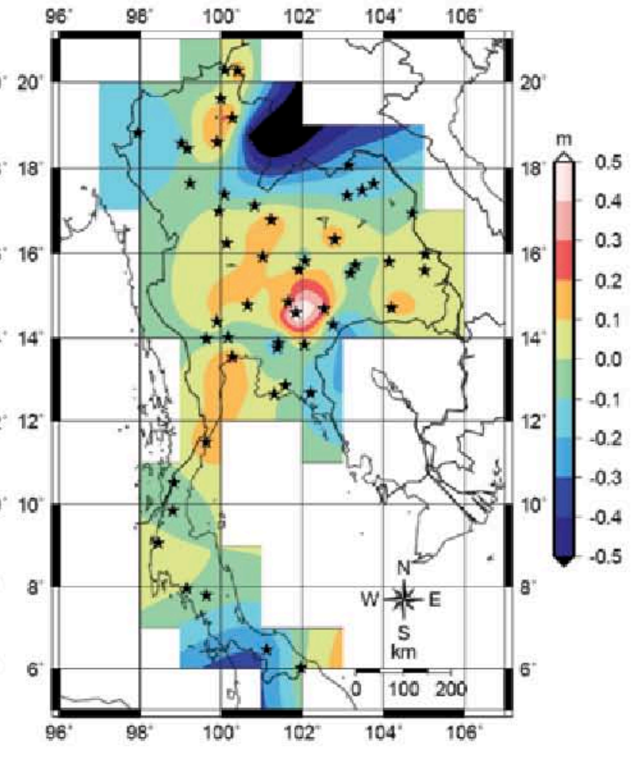

Figure 5: Differences of 53 GPS/leveling check points (star dots) and THAI12H (units in m)

5. Comparisons of THAI12G and THAI12H with EGM2008

The geoid undulations, N_M, generated from EGM2008 spherical harmonic coefficients to degree 2190 and order 2159, have the shortest wavelength of 10 arcminutes (~18 km), corresponding to the spatial resolution of 5 arcminutes (~ 9 km). Thus, if the N_M undulations are interpolated to a 30 arcsecond × 30 arcsecond grid, the information contents of smaller features than a 5 arcminute × 5 arcminute grid may not be represented by EGM2008. Figure 6a shows a plot of the differences, THAI12G minus EGM2008. The values of the differences range from −20 cm to +20 cm. The large values mostly appear in mountainous areas in north and west parts of the country, where the topography is rough and RTM data are used for modeling high-frequency signals of gravity field.

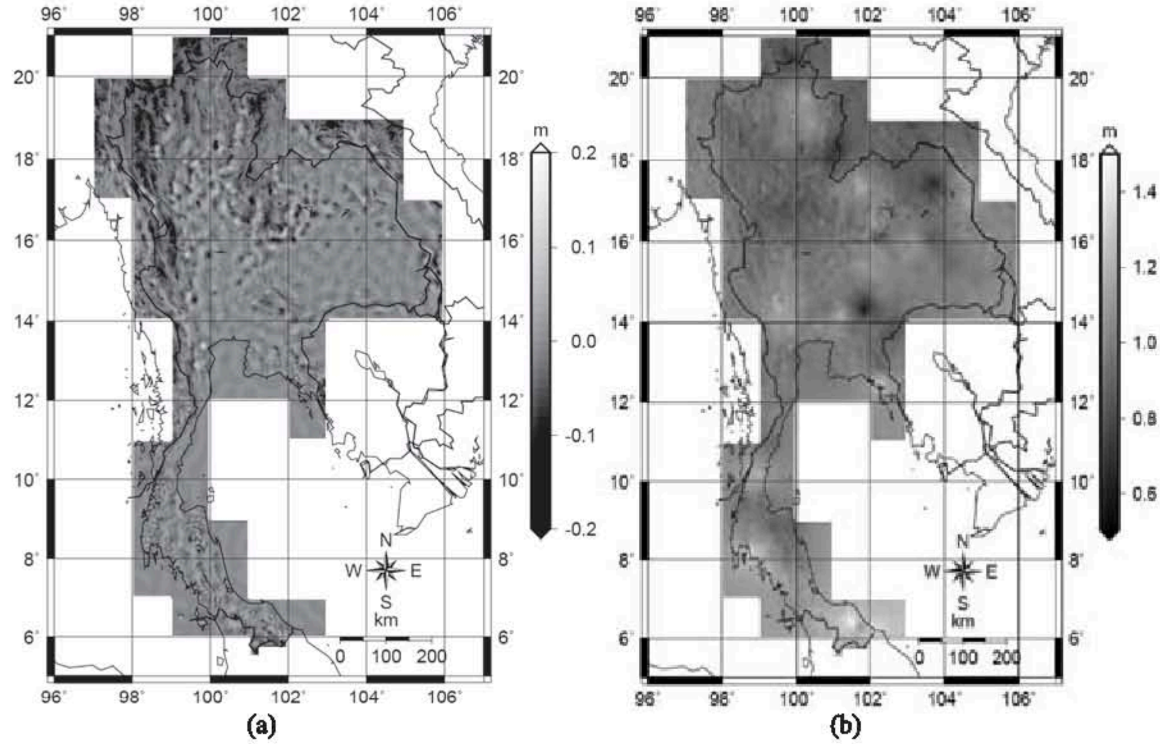

Figure 6: Differences between geoid models: (a) THAI12G and EGM2008 and (b) THAI12H and EGM2008 (units in m)

These disagreements reveal errors in terrestrial gravity data, the lack of high-frequency data, and omission errors in EGM2008. However, in Figure 6b, the differences, THAI12H minus EGM2008, vary from +50 to +150 cm, which reflect (locally) datum distortions and a predominant tilt plane in the southernmost area of the country.

6. Conclusions

A gravimetric (geocentric) geoid model, i.e. THAI12G, was computed by means of 1-D spherical FFT, from 3,949 terrestrial gravity data, SRTM digital elevation model, and EGM2008 geopotential model. The model was validated through a comparison with 200 GPS (ITRF2005) benchmarks with Kolak-1915 orthometric heights, and showed a standard deviation of ±15.1 cm (rms = 60.6 cm), with a mean-bias of +58.6cm. A 0.121-ppm tilted plane appeared in north-south direction. The comparison also revealed large discrepancies between THAI12G and the GPS/leveling points in some areas, especially, mountainous areas, where the values of differences ranged from +8.7 to +119.6 cm. A simple Gaussian covariance function was used to fit the empirical covariance function from the detrended residuals with a correlation length of 30 km. Least-squares collocation was used to produce a detrended residual surface. After a trend surface was restored and added into this residual surface, the final conversion surface was obtained. The THAI12G model was subtracted by the conversion surface to provide the final geoid model, THAI12H. The THAI12H model was a preliminary hybrid geoid model that directly related GPS ellipsoid heights and Kolak-1915 orthometric heights. The differences between THAI12H and GPS/leveling-derived geoid undulations at 53 check points had a 16.1-cm rms of fit. The results of the comparison indicated the improvement of THAI12H over EGM96 (transformed), and slightly over EGM2008 (transformed).

Although two geoid models are computed based on the use of scarce gravimetric data, particularly in mountainous terrains, they represent a significant step forward to enable us to improve them a much more accuracy for height determination. The improvement of the geoid models requires increasing and updating the number of gravity points on land and sea as well as GPS/leveling points–more intensifying gravity points and new GPS surveys are currently underway.

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